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MONOPOLES FROM QUARK CONDENSATES IN QCD*

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Chiral monopoles are hedgehoglike structures in local chiral condensates in QCD. These monopoles are (i) made of quark and gluon fields; (ii) explicitly gauge-invariant; and (iii) they carry quantized and conserved chromomagnetic charge. We argue that the chiral condensate vanishes in a core of the chiral monopole while the density of these monopoles increases with temperature wiping out the quark condensate in quark-gluon plasma. We suggest that the dynamics of the chiral monopoles is responsible for the chiral symmetry restoration in high temperature phase of QCD. We also argue that the chiral monopoles are unlikely to be associated with confining degrees of freedom. In our approach the chiral symmetry restoration and the color deconfinement in QCD are not necessarily related to each other and the corresponding transitions may occur at different temperatures.

Keywords: Chiral symmetry restoration; magnetic monopole; quark condensate

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1. Introduction

The phase structure of Quantum Chromodynamics (QCD) at non-zero temperature and finite chemical potential attracts increasing attention these days.^{1,2,3} The wide interest to the problem is motivated by intriguing chance to create a new state of matter, the quark-gluon plasma (QGP), in extraordinary hot and dense environment, which is expected to be formed in relativistic collisions of heavy nuclei.

This new state of matter is characterized by absence of the color confinement. Indeed, in the QGP the quarks and the gluons are freely propagating particles while in the hadron phase these colored degrees of freedom are tightly bound into hadrons and glueballs. Another interesting feature of hot QGP is the restoration of the chiral symmetry as the chiral condensate melts down at sufficiently high temperatures.

Clearly, both deconfining and chiral transitions must happen somewhere between hadronic and QGP phases, and they are not necessarily related to each other. Recently, lattice QCD simulations revealed that at zero baryon density the chiral

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restoration transition and the deconfinement transition may presumably occur separately: in the continuum limit the restoration of the chiral symmetry may happen at a lower temperature compared to the temperature at which the color deconfinement takes place.⁴ In realistic QCD, however, the chiral and deconfining transitions are unlikely to be real *phase* transitions. These transitions are rather smooth crossovers characterized by analytical behavior of thermodynamic quantities across transition(s), and the meaning of the transition temperature(s) is somewhat blurry. However, it seems natural to identify the temperature of the chiral transition with a temperature at which the susceptibility of a light quark condensate takes its maximum value. The deconfinement temperature can similarly be located with the help of the Polyakov loop susceptibility.⁴ The choice of these quantities is motivated by the fact that the expectation values of the quark condensate and the Polyakov loop become exact order parameters in the chiral limit and in the limit of the infinitely heavy quarks, respectively. See Ref. 4 for further review and references.

If true, the presence of the two distinct transitions in QCD – associated with the chiral symmetry restoration and with the color deconfinement – indicates that the mechanisms of the chiral symmetry breaking and the mechanism of the quark confinement may differ from each other. The confinement of quarks is usually associated with condensation of certain magnetic gluon configurations (called “Abelian monopoles”). In this mechanism of confinement – which is often called the dual superconductor scenario^a – the quark confinement is guaranteed by emergence of a confining flux tube stretched between a quark and an antiquark.^{5,6,7} The tube appears due to a dual analogue of the Meissner effect: the (chromo)electric field of the quarks is squeezed into the flux tube as a result of the condensation of the Abelian monopoles (for a review see Refs. 11, 12).

Below we discuss a chiral counterpart of the Abelian monopole, which we call a chiral monopole. We suggest that the dynamics of the chiral monopoles

- (i) causes the restoration of the chiral symmetry in the QGP phase;
- (ii) is not related, at least directly, to the color (de)confinement.

We start from a brief description of the ’t Hooft–Polyakov (HP) monopole in the Georgi–Glashow (GG) model^{13,14} (Section 2). In Section 3 we describe the chiral (or, “quark”) monopole which is a QCD analogue of the HP monopole.¹⁵ Finally, in Section 4 we discuss the mentioned link between the dynamics of the chiral monopoles and the chiral symmetry restoration at high temperature.

2. ’t Hooft–Polyakov monopoles in Georgi–Glashow model

As an illustrative example let us consider the GG model,

$$\mathcal{L}_{\text{GG}} = -\frac{1}{4}\vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} + \frac{1}{2}D_\mu\vec{\Phi} \cdot D^\mu\vec{\Phi} - \frac{\lambda}{4}[\vec{\Phi}^2 - \eta^2]^2. \quad (1)$$

^aThis scenario is related to another approach based on percolation of vortexlike magnetic structures.⁸ The Abelian monopoles and the center vortices are geometrically linked to each other.^{8,9,10}

This model describes dynamics of the $SO(3)$ gauge field A_μ^a coupled to the triplet Higgs field Φ^a , $a = 1, 2, 3$ via the adjoint covariant derivative

$$(D_\mu)^{ab} = \delta^{ab} \partial_\mu + g \epsilon^{abc} A_\mu^c, \quad (2)$$

where g is the gauge coupling and $\vec{G}_{\mu\nu} = \partial_{[\mu} \vec{A}_{\nu]} + g \vec{A}_\mu \times \vec{A}_\nu$ is strength tensor of the gauge field. The scalar coupling λ controls self-interaction of the scalar field and the condensate of the scalar field is $|\langle \vec{\Phi} \rangle| = \eta$.

The HP monopole is described by the ansatz^{13,14}:

$$\Phi^a = \frac{r^a}{g r^2} H(\eta g r), \quad A_i^a = \epsilon_{aij} \frac{r^j}{g r^2} [1 - K(\eta g r)], \quad A_0^a = 0, \quad (3)$$

where K and H are two profile functions which can be found by solving classical equations of motion of the model (1).

For a static monopole, the field $\vec{\Phi}$ has a hedgehoglike structure with respect to the spatial vector, $\vec{\Phi} \propto \vec{r}$. Since $\vec{\Phi}$ is a single-valued field then the scalar condensate in the geometrical center of the monopole should vanish, $|\vec{\Phi}(0)| = 0$. In other words, the core of the HP monopole destroys the Higgs condensate.

In quantum ensembles the behavior of the gauge field and the scalar field around an HP monopole is obviously different from the ansatz (3). Moreover, the HP monopoles are in general non-static. Thus, in order to determine the HP monopoles in nonclassical field configurations one needs a gauge- and Lorentz-invariant criterium. To this end we need to know the 't Hooft tensor¹³

$$\mathcal{F}_{\mu\nu}(n, A) = \vec{G}_{\mu\nu}(A) \cdot \vec{n} - \frac{1}{g} \vec{n} \cdot D_\mu \vec{n} \times D_\nu \vec{n}, \quad \vec{n} = \frac{\vec{\Phi}}{|\vec{\Phi}|}, \quad (4)$$

where the unit vector \vec{n} points towards the color direction of the triplet scalar field. The 't Hooft tensor (4) is the gauge-invariant field strength tensor corresponding to the composite Abelian gauge field $\mathcal{A}_\mu = \vec{A}_\mu \cdot \vec{n}$.

The monopole current can now be determined by a Maxwell equation

$$k_\nu = \frac{g}{4\pi} \partial_\mu \tilde{\mathcal{F}}_{\mu\nu}, \quad \tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \mathcal{F}_{\alpha\beta}. \quad (5)$$

By definition, the current of the HP monopole,

$$k_\nu = \int_{\mathcal{C}} d\tau \frac{\partial X_\nu^{\mathcal{C}}(\tau)}{\partial \tau} \delta^{(4)}(x - X^{\mathcal{C}}(\tau)), \quad (6)$$

has a delta-like singularity at the closed monopole worldline \mathcal{C} , parameterized by the four-vector $X^{\mathcal{C}}(\tau)$. Equations (4) and (5) guarantee that the monopoles are quantized and that the monopole charge is conserved. If one applies Eq. (5) to the HP ansatz (3) then one gets the static current $k_\mu = \delta_{\mu 0} \delta^{(3)}(\vec{r})$.

The HP monopoles have interesting dynamical and kinematical properties. These objects were first formulated as the classical solutions in the Higgs (broken) phase of the GG model.^{13,14} In this phase the non-Abelian symmetry of the model is broken down by the Higgs condensate to its Abelian subgroup, $SO(3) \rightarrow SO(2)$. The HP monopoles are rare objects in the Higgs phase (this property of the model

is consistent with the fact that the Higgs condensate is destroyed in the cores of the monopoles). However, the GG model can also reside in a symmetric (confining) phase in which the symmetry is unbroken, the Higgs condensate is absent and the density of the HP monopoles is high.

The physical picture of the phase transition from the broken (Higgs) phase into the symmetric phase can be interpreted in terms of the HP monopoles: as we move along a certain path in the coupling space starting from the broken phase towards the symmetric phase, the density of the monopoles increases^b. The Higgs condensate melts inside the monopole cores, and, as a result, the bulk expectation value of the Higgs condensate lowers with the increase of the monopole density.

At some point of our path the Higgs condensate disappears. This point corresponds to a transition separating the broken phase from the symmetric phase (i.e., at this point our path touches the boundary between the phases). The transition point corresponds to the critical density of the HP monopoles at which

- (i) the HP monopoles start to condense in a given environment;
- (ii) the color symmetry gets restored.

The symmetric phase is filled by the monopole condensate, which is absent in the Higgs phase. As we continue to move in the symmetric phase outwards the broken phase, the Higgs condensate stays zero while the monopole condensate strengthens.

We would like to apply the described monopole-mediated scenario to QCD in order to describe the chiral symmetry restoration. However, a *direct* application of the HP construction to QCD seems to be impossible^c since there are no scalar fields in QCD, and, moreover, there is no evidence that the color symmetry is broken at low baryon density. Nevertheless, monopolelike defects of a HP type can be identified in a non-Abelian gauge theory with (generally, dynamical) quarks, and, as we argue below, the dynamics of these monopoles may indeed be related to the chiral symmetry breaking/restoration.

3. Chiral monopoles in QCD

For simplicity let us consider the $SU(2)$ gauge theory with one quark field ψ which transforms in the fundamental representation of the gauge group. Generalizations to a multiflavor theory¹⁵ and to the case of the $SU(3)$ group¹⁶ are straightforward.

The key idea is to use the quark field ψ in order to construct a composite scalar field $\vec{\xi}$ transforming in the adjoint representation of the gauge group. The field $\vec{\xi}$ should then play a role which is similar to the role the scalar field $\vec{\Phi}$ in the GG

^bFor simplicity we discuss the model at zero temperature in order to avoid inessential details related to appearance of the another symmetric phase (“Coulomb phase”) which is deconfining. This Coulomb phase is also characterized by enhanced – with respect to the broken phase – density of the HP monopoles, which are, however, not condensed.

^cIn (pure) non-Abelian gauge theories one can still construct an effective Higgs field via an Abelian gauge fixing and then identify Abelian monopoles relevant to the confinement of color.^{11,12}

model. This composite field can be constructed in various ways. A simplest choice is given by the quark-antiquark bilinears, which can formally be written as:

$$\vec{\xi}_\Gamma = \bar{\psi}(x)\Gamma\vec{\tau}\psi(x), \quad \Gamma = \mathbb{1}, i\gamma_5. \quad (7)$$

Here $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices acting in the color space and γ_μ, γ_5 is the standard set of the spinor γ -matrices. The real-valued fields $\vec{\xi}_S$ and $\vec{\xi}_A$ transform with respect to the rotations/reflections of the coordinate space as the scalar field and the pseudoscalar (axial) field, respectively. The subscripts S and A correspond, respectively, to the scalar, $\Gamma = \mathbb{1}$, and axial, $\Gamma = i\gamma_5$, operators.

In order to make the definition (7) meaningful one should consider, for example, the fermion field ψ as a c -valued function. It is convenient to choose the field ψ to be an eigenmode ψ_λ of the Dirac operator $\mathcal{D} \equiv \mathcal{D}[A]$, which corresponds to a given background configuration of the gauge field A_μ :

$$\mathcal{D}[A]\psi_\lambda(x) = \lambda\psi_\lambda(x), \quad \mathcal{D}[A] = \gamma_\mu(\partial_\mu + i\frac{1}{2}\tau^a A_\mu^a) + m, \quad (8)$$

The Dirac eigenmodes are labeled by the eigenvalues λ of the Dirac operator, so that one can identify infinite number of the effective adjoint composite fields corresponding to a given background gauge field A ,

$$\vec{\xi}_{\Gamma,\lambda} = \bar{\psi}_\lambda(x; A)\Gamma\vec{\tau}\psi_\lambda(x; A). \quad (9)$$

The eigenvalue index λ of the composite fields (9) corresponds to a virtual energy scale which is “resolved” by the composite field $\vec{\xi}_{\Gamma,\lambda}$.

One can also define the composite scalar field $\vec{\xi}$ as an average of a quark-antiquark bilinear over all possible fermion fields in the background of the given gauge field \vec{A}_μ . This composite field can be represented a (local) quark condensate

$$\vec{\xi}_\Gamma^{(\psi)}[A] = \langle \bar{\psi}(x)\Gamma\vec{\tau}\psi(x) \rangle_A \equiv \sum_\lambda \frac{\bar{\psi}_\lambda(x; A)\Gamma\vec{\tau}\psi_\lambda(x; A)}{\lambda - im}. \quad (10)$$

To our mind this equation represents the most suitable choice of the composite adjoint scalar field. First of all, the composite field (10) is written in a simple and natural way. There is no predistinguished energy scale λ labeling this field. Finally, the nonperturbative (infrared) eigenmodes enter Eq. (10) with a higher weight compared to the perturbative (ultraviolet) modes. For shortness, we use below the notation $\vec{\xi}_\Gamma$ for both definitions (9) and (10).

Under the axial transformations (α is the global parameter of the axial rotation),

$$U_A(1): \quad \psi \rightarrow e^{i\alpha\gamma_5}\psi, \quad \bar{\psi} \rightarrow \bar{\psi}e^{i\alpha\gamma_5}, \quad (11)$$

the color vectors $\vec{\xi}_S$ and $\vec{\xi}_A$ transform via each other:

$$\begin{pmatrix} \vec{\xi}_S \\ \vec{\xi}_A \end{pmatrix} \rightarrow \begin{pmatrix} \vec{\xi}_S \\ \vec{\xi}_A \end{pmatrix}' = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \vec{\xi}_S \\ \vec{\xi}_A \end{pmatrix}. \quad (12)$$

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Using two adjoint fields ξ_Γ we can define three unit color vectors

$$\vec{n}_S = \frac{\vec{\xi}_S}{|\vec{\xi}_S|}, \quad \vec{n}_A = \frac{\vec{\xi}_A}{|\vec{\xi}_A|}, \quad \vec{n}_I = \frac{\vec{\xi}_S \times \vec{\xi}_A}{|\vec{\xi}_S \times \vec{\xi}_A|}. \quad (13)$$

Here (\vec{u}, \vec{v}) and $[\vec{u} \times \vec{v}]^a = \epsilon^{abc} u^b v^c$ are, respectively, the scalar and the vector products in the color space of the vector \vec{u} and \vec{v} , and $|\vec{u}| = (\vec{u}, \vec{u})^{1/2}$ is the norm of the color vector \vec{u} . The vector \vec{n}_I is invariant under the axial transformations (12) because it is a (normalized) vector product of the scalar and axial vectors.

Our definition of the composite adjoint fields $\vec{\xi}_\Gamma$ inherently “locks” the axial rotations with the gauge rotations since Eq. (12) may also be regarded as a global rotation in a color (gauge group) space by the angle 2α around the color direction \vec{n}_I .

Now we interpret the unit vectors (13) as directions of certain composite adjoint Higgs fields. We have three sets of the fields $\{\vec{n}_\Gamma, \vec{A}_\mu\}$ with $\Gamma = S, A, I$, which can be used to construct three gauge invariant ’t Hooft tensors (4) as it was already done in Section 2 for the case of the GG model:

$$\mathcal{F}_{\mu\nu}^\Gamma(n_\Gamma, A) = F_{\mu\nu}^a(A) n_\Gamma^a - \frac{1}{g} \epsilon^{abc} n_\Gamma^a (D_\mu^{\text{ad}} n_\Gamma)^b (D_\nu^{\text{ad}} n_\Gamma)^c, \quad \Gamma = S, A, I. \quad (14)$$

The ’t Hooft tensor (14) is the gauge-invariant field strength tensor for the diagonal (with respect to the color direction \vec{n}_Γ) component of the gauge field,

$$\mathcal{A}_\mu^\Gamma = A_\mu^a n_\Gamma^a, \quad \Gamma = S, A, I. \quad (15)$$

We now come close to the definition of the chiral monopole(s). The current of the chiral monopole of the Γ -th type is

$$k_\nu^\Gamma = \frac{g}{4\pi} \partial_\mu \tilde{\mathcal{F}}_{\mu\nu}^\Gamma, \quad (16)$$

where we used Eq. (5) derived in the GG model. The currents (16) have delta-like singularities at the corresponding worldlines. The monopole charges of the chiral monopoles – defined according to Eq. (16) – are quantized and conserved. We would like to stress that the chiral monopoles in QCD are explicitly gauge invariant.

According to Eq. (15) the chiral monopoles of the Γ -th type carry the magnetic charges with respect to the “scalar” ($\Gamma = S$), “axial” ($\Gamma = A$) and “chirally invariant” ($\Gamma = I$) components of the gauge field \vec{A}_μ . In the corresponding Unitary gauges, $n_\Gamma^a = \delta^{a3}$, the quark monopoles correspond to monopoles “embedded” into the diagonal component (15). In the gauges, where the diagonal component \mathcal{A}_μ^Γ is regular, such monopoles are hedgehogs in the composite quark-antiquark fields (colored quark condensates). Each monopole is characterized by the typical hedgehoglike behavior ($\vec{n}_\Gamma \sim \vec{r}$ for static monopoles) in the local vicinity of the monopole core. The very existence of these monopoles in QCD is not a dynamical fact but rather a simple kinematical consequence of the existence of the adjoint real-valued fields defined via Eqs. (7) and (13).

Thus, the chiral monopole is gauge-invariant hedgehoglike structure in the “colored quark condensate” (9) or (10). The composite nature of the condensate does

not undermine the existence of the chiral monopoles. For example, a very similar structure, called the (embedded) Nambu monopole, is a well known field defect in the Standard Electroweak theory.¹⁷ In the Electroweak theory the role of the adjoint composite field (7) is played by the scalar triplet $\vec{\xi}_{EW} = \Phi^\dagger \vec{\tau} \Phi$ field, where Φ is the two-component Higgs field. A similar type of defects exists also in a superfluid Helium as well as in certain types of liquid crystals.¹⁸

4. Chiral monopoles and chiral symmetry restoration

Having in mind the analogy between the chiral monopole in QCD and the HP monopole in the GG model, one can suggest that the properties of the chiral monopoles are related to the restoration of the chiral symmetry in the high-temperature phase of QCD.¹⁵ The chain of considerations is as follows.

Firstly, the cores of the chiral monopoles should contain a chirally symmetric vacuum (this statement is intuitively clear because of the hedgehoglike structure of the local condensates).¹⁵ Secondly, it was found numerically that the density of the chiral monopoles increases with temperature.¹⁶ One can interpret this observation as a destruction of the bulk expectation value of the chiral condensate by the cores of the chiral monopoles: as density of the chiral monopole increases, the vacuum of QCD gradually turns from the chirally broken phase to the chirally symmetric phase. Thirdly, another numerical argument in favor of our conclusion can be found with the help of the mode-by-mode analysis: the density of the chiral monopoles is anti-correlated with the density of the Dirac eigenmodes (the lower density of the Dirac eigenmodes the higher monopole density).¹⁶

Most probably the chiral monopoles are not related to the confining properties of QCD. Indeed, the confinement of color needs a certain amount of disorder in the gauge fields. The disorder is usually reflected in existence of a kind of a condensate made of percolating defect trajectories. In other words, the defects, which presumably cause the confinement, are to be propagating (proliferating) for infinitely long distances in the confinement phase. The condensate of the defects must disappear in the deconfinement phase. These properties were indeed observed in $SU(2)$ Yang–Mills theory both for the Abelian monopoles^{11,12} and for the center vortices⁸, both of which are the most probable candidates for the confining gluonic configurations. However, the chiral monopoles are suggested to be percolating in the chirally symmetric QGP phase which is, however, not confining.¹⁵ On the contrary, in the QGP phase the Abelian monopoles may form a gaseous or a liquid state, rather than a condensate.¹⁹ Thus, the properties of the confining (Abelian) monopoles and the chiral monopoles are rather different.

The chiral monopole is an object made of both quark and gluon fields. Therefore, one can expect that the presence of the chiral monopole affects not only the quark condensate in the local vicinity of the monopole, but the chiral monopole may also have its imprint on the gluonic fields (in a complete analogy with the Abelian monopoles, see Refs. 20, 21 for details). Indeed, it was found numerically in Ref. 16

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that the chiral monopoles possess gluonic cores: on average, the chromomagnetic energy near the monopole trajectories is higher compared to the chromomagnetic energy far from the monopole cores. Features of the gluonic core are consistent with the asymptotic freedom. The clear gluonic structure of the chiral monopole in QCD makes this monopole very similar to the HP monopole in the GG model.

Summarizing, we suggested a scenario of the chiral symmetry restoration in the QGP phase. We argued that the increase of the density of the chiral monopoles with temperature leads to the gradual suppression of the chiral condensate in this phase. At the same time, the chiral monopoles seem to be unrelated to the confinement of color, so that the chiral symmetry restoration and the deconfinement transition may occur at different temperatures.

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